Baltic Olympiad in Informatics
27 Apr – 2 May, 2019
Tartu, Estonia

Day: **2**
Task: **kitchen**
Version: **en-1.0**

Tom’s Kitchen

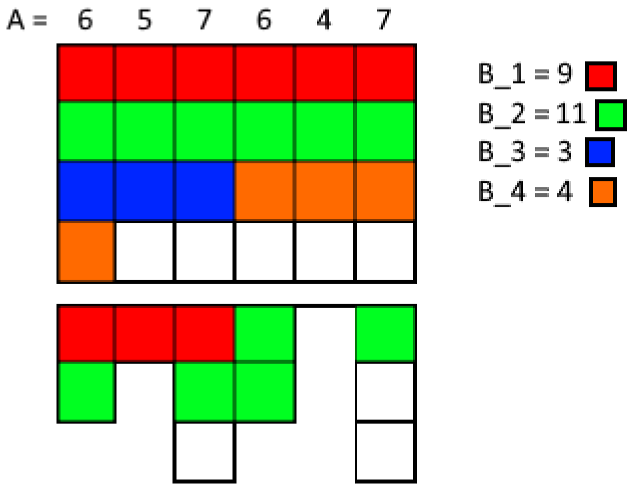
Spoiler

Subtask 1 was intended to permit simple case-analysis based solutions.

Subtask 2 was intended to permit brute force search solutions.

Subtask 3 was a reduction to a standard Dynamic Programming problem (commonly stated as: you have certain coins, find if you can pay exactly X amount of money with them).

Subtask 4 was intended to permit task-specific but suboptimal Dynamic Programming solutions.



The full solution uses Dynamic Programming. Let us mentally reorder the hours spent on each meal such that for the first K hours, all chefs are different. This way we can visualize these K hours forming an $N \times K$ ”diversity box”, with all ”non-diverse hours” coming afterwards (as shown in the figure above). Now let’s make the following observations:

1. Each chef can add at most 1 hour to any column of the ”diversity box”.
2. A chef j can fill the ”diversity box” by at most $\min(B_j, N)$.
3. In a correct solution ”diversity box” must be filled by an amount at least $N \cdot K$.
4. Suppose you have decided to hire chefs for a total of H hours. Then it’s optimal to hire such set of chefs that the ”diversity box” is filled as much as possible (perhaps even overfilled).

Now let $D[c][h]$ be the maximum amount we can fill the ”diversity box” by picking a subset of chefs $1, \ldots, c$ such that they are hired for a total of h hours. Now let us notice that for the value $D[c][h]$ there are two possibilities:

1. The maximal subset contains chef c. Thus the other chefs in this subset form a maximal solution for $D[c-1][h-B_c]$ (otherwise we could pick a better subset). Thus $D[c][h] = D[c-1][h-B_c] + \min(B_c, N)$.
2. The maximal subset doesn’t contain chef c. Thus this subset also forms a maximal solution for $D[c-1][h]$ (a better solution to $D[c-1][h]$ would contradict the maximality of this subset). Thus $D[c][h] = D[c-1][h]$.

Now we simply need to consider the two cases and see which gives us a better solution. Thus $D[c][h] = \max(D[c-1][h-B_c]+\min(B_c, N), D[c-1][h])$. For performing the dynamic programming computation, we can initialize $D[0][0]$ to 0 and every other $D[i][j]$ to ∞. Note that once we have computed $D[c][*]$ we don't care about $D[c-1][*]$ anymore, so we can optimize memory consumption. The answer will be minimum non-negative $h - \sum_i A_i$ such that $D[N][h] \geq N \cdot K$.

```
import array
N = 300
n,m,k = [int(x) for x in input().split()]
a = [int(x) for x in input().split()]
b = [int(x) for x in input().split()]
supply = array.array(’i’, [−N∗N for i in range(N∗N+1)])
supply[0] = 0;
def solve():
    if(min(a) < k):
        return ’Impossible’
    bsum = 0
    for x in b:
        bsum += x
        for i in range(bsum,−1,−1):
            supply[i+x] = max(supply[i+x],supply[i]+min(x,n))
    for i in range(sum(a),N∗N+1):
        if(supply[i] >= n∗k):
            return i−sum(a)
    return ’Impossible’
print(solve())
```

Credits

- Task: Bernhard Linn Hilmarsson (Iceland)
- Solutions and tests: Oliver-Matis Lill, Andres Unt (Estonia)